Exercise 1

Solve the differential equation.

$$4y'' - y = 0$$

Solution

This is a linear homogeneous ODE with constant coefficients, so it has solutions of the form $y = e^{rx}$.

$$y = e^{rx} \quad \rightarrow \quad y' = re^{rx} \quad \rightarrow \quad y'' = r^2 e^{rx}$$

Substitute these formulas into the ODE.

$$4(r^2 e^{rx}) - e^{rx} = 0$$

Divide both sides by e^{rx}.

$$4r^2 - 1 = 0$$

Solve for r.

$$(2r + 1)(2r - 1) = 0$$

$$r = \left\{-\frac{1}{2}, \frac{1}{2}\right\}$$

Two solutions to the ODE are $e^{-x/2}$ and $e^{x/2}$. According to the principle of superposition, the general solution to the ODE is a linear combination of these two.

$$y(x) = C_1 e^{-x/2} + C_2 e^{x/2}$$